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Oxford Cambridge and RSA

A Level Mathematics A
H240/03 Pure Mathematics and Mechanics
Sample Question Paper

Date – Morning/Afternoon   Version 2.1
Time allowed: 2 hours

You must have:
• Printed Answer Booklet

You may use:
• a scientific or graphical calculator

INSTRUCTIONS

• Use black ink. HB pencil may be used for graphs and diagrams only.
• Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
• Answer **all** the questions.
• **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
• Do **not** write in the bar codes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
• The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

• The total number of marks for this paper is **100**.
• The marks for each question are shown in brackets **[ ]**.
• **You are reminded of the need for clear presentation in your answers.**
• The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r} \quad \text{for } |r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1\,a^{n-1}b + {}^nC_2\,a^{n-2}b^2 + \ldots + {}^nC_r\,a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where $\displaystyle {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \left(|x| < 1,\ n \in \mathbb{R}\right)$

Differentiation

f(x)	f$'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle \int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$

$\displaystyle \int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,dx = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle \int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B) \quad$ **or** $\quad P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n,\, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu,\, \sigma^2)$ then $\bar{X} \sim N\left(\mu,\, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,\, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions

1   **(a)**   If $|x| = 3$, find the possible values of $|2x-1|$. **[3]**

(b)   Find the set of values of x for which $|2x-1| > x+1$.
Give your answer in set notation. **[4]**

2   **(a)**   Use the trapezium rule, with four strips each of width 0.25, to find an approximate

value for $\displaystyle\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$. **[3]**

(b)   Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**

3   **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0° < x < 90°$, find the exact value of $\sin x$. **[4]**

4   For a small angle θ, where θ is in radians, show that $1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$. **[4]**

5 **(a)** Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x. **[3]**

(b) The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+....$.

Find the possible values of the constants p and q. **[5]**

6 A curve has equation $y=x^2+kx-4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x=-2$

- find the value of k

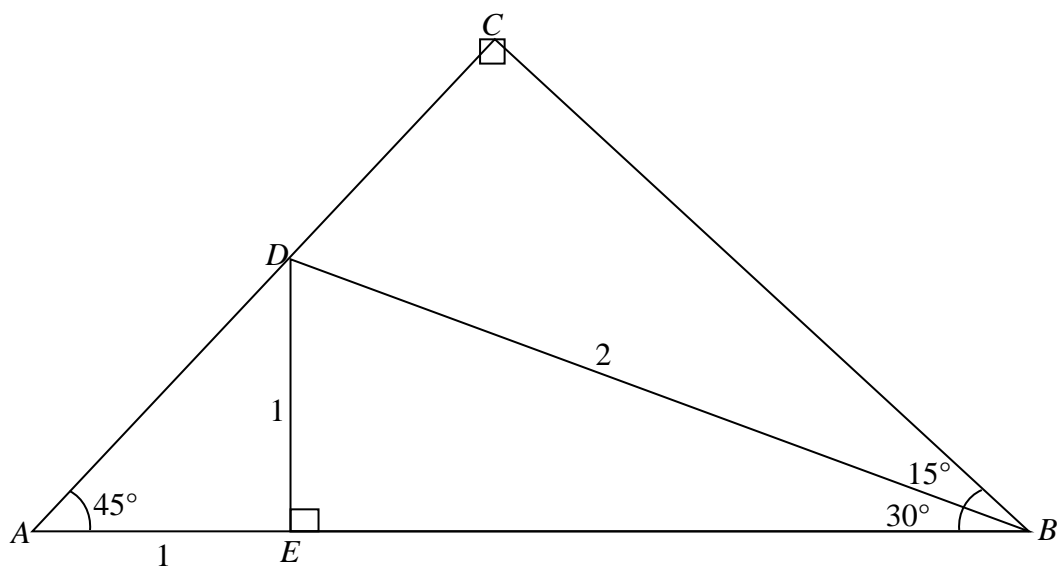- show that the curve has a point of inflection which is not a stationary point. **[7]**

7 **(a)** Find $\int 5x^3\sqrt{x^2+1}\,dx$. **[5]**

(b) Find $\int \theta \tan^2\theta\,d\theta$.

You may use the result $\int \tan\theta\,d\theta = \ln|\sec\theta|+c$. **[5]**

8 **In this question you must show detailed reasoning.**

The diagram shows triangle *ABC*.

The angles *CAB* and *ABC* are each $45°$, and angle $ACB = 90°$.

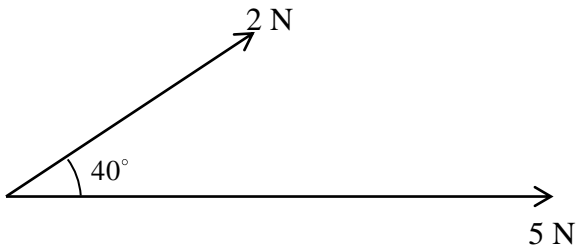The points *D* and *E* lie on *AC* and *AB* respectively. $AE = DE = 1$, $DB = 2$.

Angle $BED = 90°$, angle $EBD = 30°$ and angle $DBC = 15°$.

(a) Show that $BC = \dfrac{\sqrt{2} + \sqrt{6}}{2}$. **[3]**

(b) By considering triangle *BCD*, show that $\sin 15° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$. **[3]**

Section B: Mechanics
Answer **all** the questions

9 Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.

(a) Calculate the magnitude of the resultant force on the particle. **[3]**

(b) Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

10 A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal.
The body is held at rest on the plane by the action of a force of magnitude P N.
The force is acting up the plane in a direction parallel to a line of greatest slope of the plane.
The coefficient of friction between the body and the plane is μ.

(a) When $P = 100$, the body is on the point of sliding down the plane.

Show that $g \sin \alpha = g \mu \cos \alpha + 5$. **[4]**

(b) When P is increased to 150, the body is on the point of sliding up the plane.

Use this, and your answer to part **(a)**, to find an expression for α in terms of g. **[3]**

11 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time t seconds is given by
$$\mathbf{r} = 2t^3\mathbf{i} + \left(5t^2 - 4t\right)\mathbf{j}.$$

(a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately $044°$.

[3]

(b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$.

[4]

(c) Determine the times at which the particle is moving on a bearing of $045°$. **[2]**

12 A girl is practising netball.

She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.

The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.
- The initial velocity of the ball has magnitude U m s^{-1}.
- The angle of projection is $40°$.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.
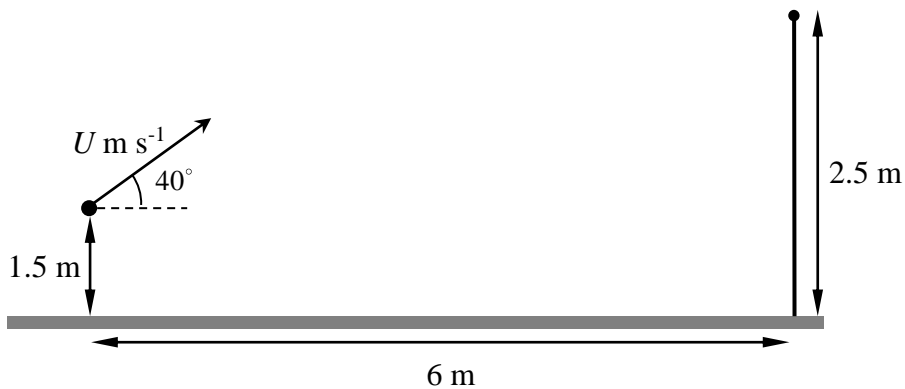
(a) For $U = 10$, find

    (i) the greatest height above the ground reached by the ball **[5]**

    (ii) the distance between the ball and the hoop when the ball is vertically above the hoop. **[4]**

(b) Calculate the value of U which allows her to hit the hoop. **[3]**

(c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

(d) Suggest one improvement that might be made to this model. **[1]**

13 Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1} \frac{3}{4}$ to the horizontal.

Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1} \frac{4}{3}$ to the horizontal.

The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P.

The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth.

Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.
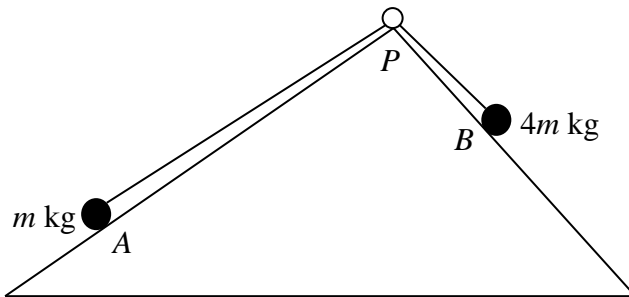
(a) Show that when A is released it accelerates towards the pulley at $\dfrac{7g}{15}$ m s^{-2}. **[6]**

(b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its

speed is $\sqrt{\dfrac{7g}{30}}$ m s^{-1}. **[2]**

14 A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall.

The ladder is inclined at an angle of $45°$ to the horizontal.

A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A.

The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction

between the ladder and the ground is $\frac{1}{2}$.

The system is in limiting equilibrium.

Find x. **[8]**

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END OF QUESTION PAPER

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